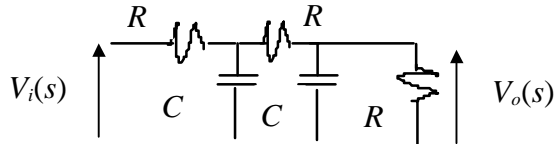


**Ph.D. Qualifying Exam
Control Systems
Sample**

I. Modeling

- a) Write the node equations governing the circuit then obtain the transfer function $G(s) = V_o(s)/V_i(s)$ using Cramer's rule.



- b) Write state-space equations for the circuit using the transfer function of (a). What is the physical significance of the state variables? Give another set of physical state variables for the circuit that has a clear physical meaning.

II. Analysis

- a) Determine the closed-loop stability conditions for the plant

$$G(s) = \frac{s + 2}{s(s + a)(s - 5)}$$

with the PD control

$$C(s) = K(s + 1)$$

What is the stable range of K for the nominal values $a = 0$ and $a = 1$? Verify your answer by sketching the system root locus in each case. Find the asymptotes, the break-in and breakaway points, and the $j\omega$ -axis intercept for the root locus.

- b) Obtain the Nyquist plot for the system using MATLAB and show the gain margin and phase margin on your plot.

$$G(s) = \frac{1}{s(s + 1)(s/5 + 1)}$$

- c) Determine the stability of the difference equation

$$x(k+2) - 0.5x(k+1) + 2x(k) = u(k+1)$$

III. Design

- a) Design a controller for the plant of II(b) to obtain
- i) Zero steady-state error due to unit step input.
 - ii) A settling time less than 1.7 s.
 - iii) Percentage overshoot less than 2%

Check the system time response including the frequency of oscillations, the actual overshoot and settling time and the time to first peak using MATLAB and comment on the design.

- b) Determine the bandwidth of the closed-loop system by sketching its Bode plot. What is the significance of the bandwidth? Explain using results from (a).